Exercise 19B: Solutions

1 a $x = 2t^2 - 6t + c$

When $t = 0, x = 0$.

$$\therefore \quad 0 = 0 - 0 + c$$
$$c = 0$$
$$x = 2t^2 - 6t$$

b $t = 3$

$$x = 2 \times 3^2 - 6 \times 3$$
$$= 0$$

It will be at the origin, O.

c Consider when $v = 0$:

$$4t - 6 = 0$$
$$t = \frac{3}{2}$$
$$x = 2 \times \left(\frac{3}{2}\right)^2 - 6 \times \frac{3}{2}$$
$$= -4\,\frac{1}{2}$$

The particle will travel $4\,\frac{1}{2}$ cm to the left of the origin and back, for a total of 9 cm.

d Average velocity

$$= \frac{\text{change in position}}{\text{change in time}}$$
$$= \frac{0}{3} = 0 \text{ cm/s}$$

e $$\text{Average speed} = \frac{\text{distance travelled}}{\text{change in time}}$$
$$= \frac{9}{3} = 3 \text{ cm/s}$$

2 a $x = t^3 - 4t^2 + 5t + c$

When $t = 0, x = 4$.

$$\therefore \quad 4 = 0 - 0 + 0 + c$$
$$c = 4$$
$$x = t^3 - 4t^2 + 5t + 4$$
$$a = \frac{dv}{dt}$$
$$= 6t - 8$$

b $$3t^2 - 8t + 5 = 0$$
$$(3t - 5)(t - 1) = 0$$
$$t = \frac{5}{3} \text{ or } 1$$

When $t = \frac{5}{3}$,

$$x = \left(\frac{5}{3}\right)^3 - 4 \times \left(\frac{5}{3}\right)^2 + 5 \times \frac{5}{3} + 4$$
$$= \frac{125}{27} - \frac{100}{9} + \frac{25}{3} + 4$$
$$= 5\,\frac{23}{27}$$

When $t = 1$,

$$x = 1^3 - 4 \times 1^2 + 5 \times 1 + 4$$
$$= 6$$

c When $t = \frac{5}{3}$,

$$a = 6 \times \frac{5}{3} - 8$$
$$= 2 \text{ m/s}^2$$

When $t = 1$,

$$a = 6 \times 1 - 8$$
$$= -2 \text{ m/s}^2$$

3

$$v = 10t + c$$
$$x = 5t^2 + ct + d$$

When $t = 2:$

$$x = 5 \times 2^2 + 3c + d = 0$$
$$2c + d = -20 \quad (1)$$

When $t = 3:$

$$x = 5 \times 3^2 + 3c + 2 = 25$$
$$3c + d = -20 \quad (2)$$

$(2) - (1): c = 0$

$$d = -20$$
$$x = 5t^2 - 20$$

When $t = 0, x = -20$

20 m to the left of O

4 $a = 2t - 3$

$v = t^2 - 3t + c$

When $t = 0, v = 3.$

$$3 = 0 - 0 + c$$
$$c = 3$$
$$v = t^2 - 3t + 3$$
$$x = \frac{t^3}{3} - \frac{3t^2}{2} + 3t + d$$

When $t = 0, x = 2.$

$$2 = 0 - 0 + 0 + d$$
$$d = 2$$
$$x = \frac{t^3}{3} - \frac{3t^2}{2} + 3t + 2$$

When $t = 10$,

$$x = \frac{10^3}{3} - \frac{3 \times 10^2}{2} + 3 \times 10 + 2$$
$$= \frac{2000 - 900}{6} + 32$$
$$= 215\frac{1}{3} \text{m}$$
$$v = t^2 - 3t - 3$$
$$= 10^2 - 3 \times 10 + 3$$
$$= 73 \text{m/s}$$

5 a $a = -10$

$v = -10t + c$

When $t = 0, v = 25$.

$25 = 0 + c$

$c = 25$

$v = -10t + 25$

b $v = -10t + 25$

$x = -5t^2 + 25t + d$

When $t = 0, x = 0$.

(Define the point of projection as $x = 0$, the origin.)

$0 = 0 + 0 + d$

$d = 0$

$x = -5t^2 + 25t$

c Maximum height occurs when $v = 0$.

$v = -10t + 25 = 0$

$t = \frac{25}{10} = \frac{5}{2}$

2.5 s after projection

d When $t = 2.5$,

$x - 5t^2 + 25t$

$= -5 \times 2.5^2 + 25 \times 2.5$

$= 31.25 \text{ m}$

e $x = -5t^2 + 25t = 0$

$-5t(t - 5) = 0$

$t = 5 \text{ (} t = 0 \text{ is the start)}$

6 Define $t = 0$ as the moment the lift passes the 50th floor.

$a = \frac{1}{9}t - \frac{5}{9}$

$v = \frac{1}{18}t^2 - \frac{5}{9}t + c$

$-8 = 0 - 0 + c$

$c = -8$

$v = \frac{1}{18}t^2 - \frac{5}{9} - 8$

$x = \frac{1}{54}t^3 - \frac{5}{18}t^2 - 8t + d$

$50 \times 6 = 0 - 0 - 0 + d$

$d = 300$

$v = 0$ when

$\frac{1}{18}t^2 - \frac{5}{9}t - 8 = 0$

$t^2 - 10t - 8 \times 18 = 0$

$(t - 18)(t + 8) = 0$

$t = 18$

$x = \frac{1}{54}t^3 - \frac{5}{18}t^2$

$- 8t + 300$

$= \frac{1}{54} \times 18^3 - \frac{5}{18}18^2$

$$- 8t + 300$$

$$= \frac{1}{54} \times 18^3 - \frac{5}{18} 18^2$$

$$- 8 \times 18 + 300$$

$$= 174$$

$$\frac{174}{6} = 29$$

It will stop on the 29th floor.